Formulas and Equations for Ideal Gas and Kinetic Theory of Gases

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Ideal Gas Law

The ideal gas law relates the pressure (P), volume (V), and temperature (T) of an ideal gas.

$$PV = nRT \tag{1}$$

where:

- P = Pressure of the gas (in Pascals, Pa)
- V = Volume of the gas (in cubic meters, m^3)
- n = Number of moles of the gas
- R = Universal gas constant ($8.314\,\text{J/mol·K}$)
- T = Absolute temperature (in Kelvin, K)

Ideal Gas Law in Terms of Boltzmann Constant

The ideal gas law can also be expressed using the Boltzmann constant (k).

$$PV = NkT \tag{2}$$

where:

- N = Number of gas molecules
- k = Boltzmann constant ($1.38 \times 10^{-23}\,\text{J/K}$)

Density of an Ideal Gas

The density (ρ) of an ideal gas can be derived from the ideal gas law.

$$PM = \rho RT \tag{3}$$

where:

- M = Molar mass of the gas (in kg/mol)
- ρ = Density of the gas (in kg/m^3)

Mean Free Path

The mean free path (λ) is the average distance a gas molecule travels between collisions.

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n} \tag{4}$$

where:

- d = Diameter of the gas molecule (in meters, m)
- n = Number density of molecules (in molecules/m^3)

Speed of Gas Molecules

The speed of gas molecules can be described using the following formulas:

Root Mean Square Speed (v_{rms})

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}} \tag{5}$$

where:

- m = Mass of a single gas molecule (in kg)
- M = Molar mass of the gas (in kg/mol)

Average Speed (v_{avg})

$$v_{\text{avg}} = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}} \tag{6}$$

Most Probable Speed (v_{mp})

$$v_{\text{mp}} = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2RT}{M}} \tag{7}$$

Degree of Freedom

The degree of freedom (f) is the number of independent ways a molecule can store energy.

- For a monatomic gas: $f = 3$ (translational only)
- For a diatomic gas: $f = 5$ (3 translational + 2 rotational)
- For a polyatomic gas: $f = 6$ (3 translational + 3 rotational)

Molar Specific Heats

The molar specific heats at constant volume (C_V) and constant pressure (C_P) are related to the degree of freedom.

Molar Specific Heat at Constant Volume (C_V)

$$C_V = \frac{f}{2}R \tag{8}$$

Molar Specific Heat at Constant Pressure (C_P)

$$C_P = C_V + R = \left(\frac{f}{2} + 1\right) R \tag{9}$$

Ratio of Specific Heats (γ)

$$\gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f} \tag{10}$$

Kinetic Energy of Gas Molecules

The average kinetic energy of a gas molecule is given by:

$$\text{Kinetic Energy} = \frac{3}{2}kT \tag{11}$$

Maxwell-Boltzmann Distribution

The Maxwell-Boltzmann distribution describes the distribution of speeds of gas molecules.

$$f(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}} \tag{12}$$

where:

- $f(v)$ = Probability density function
- v = Speed of the gas molecule